Universität Konstanz

Real Geometry and Algebra Seminar in Honor of Claus Scheiderer's 65th Birthday

17th May 2024, 9:00AM - 4:00PM in G309

Programme and Book of Abstracts

Time	Speaker	Title
9:00	D. Plaumann	Sums of squares – themes and variations in Claus's work
10:00	K. Becher	On the u-invariant of a function field
11:00	Coffee Break	
11:30	G. Averkov	Convex hulls of monomial curves and a sparse positivstellensatz
12:30	Lunch Break	
14:00	G. Blekherman	Bounds on Pythagoras number
15:15	M. Kummer	Spectrahedral Shadows and Completely Positive Maps on Real Closed Fields
19:00	Dinner	

The Dinner takes place in

1. Daniel Plaumann (TU Dortmund). Sums of squares – themes and variations in Claus's work.

Sums of squares of polynomials, and the special role they play in real algebraic geometry and its applications, have been and continue to be at the heart of Claus's research. I will not attempt to survey his work, but will instead focus on three recurring themes and a few key results.

2. Karim Becher (Universty of Antwerp). On the u-invariant of a function field.

The u-invariant of a field is the largest dimension of an anisotropic qua-

dratic torsion form over the field. This field invariant was introduced by I. Kaplansky in 1953 and addapted to the study of real fields by R. Elman and T.Y. Lam in 1973. In 2009, D. Harbater, J. Hartmann, and D. Krashen obtained a bound on the u-invariant for nonreal function fields in one variable over a complete discretely valued field. This was extended by C. Scheiderer to cover the case real function fields.
In a joint work with N. Daans and V. Mehmeti, we obtain a more general version of this result, namely for function fields of curves over a henselian valued field with arbitrary value group.

3. Gennadiy Averkov (TU Cottbus-Senftenberg). Convex hulls of monomial curves and a sparse positivstellensatz.

Consider the closed convex hull K of a monomial curve given parametrically as $(t^(m_1), \ldots, t^(m_n))$, with the parameter t varying in an interval I. We show, using constructive arguments, that K admits a lifted semidefinite description by $O(d)$ linear matrix inequalities (LMIs), each of size at most $n/2+1$, where d is the degree of the curve. On the dual side, we show that if a univariate polynomial $p(t)$ of degree d with at most $2k+1$ monomials is non-negative on R_+, then p admits a representation $p = t^0 s_0 + + t^(dk) s_k$, where the polynomials $s_0, \ldots, s_(dk)$ are sums of squares of degree at most $2k$. The latter is a univariate positivstellensatz for sparse polynomials, with non-negativity of p being certified by sos polynomials whose degree only depends on the sparsity of p. Our results fit into the general attempt of formulating polynomial optimization problems as semidefinite problems with LMIs of small size. Such small-size descriptions are much more tractable from a computational viewpoint. (Joint work with Claus Scheiderer).

4. Greg Blekerman (Georgia Tech). Bounds on Pythagoras number.

Given number of variables n and even degree $2d$, Pythagoras number is the least k, such that any sum of squares in n variables of degree 2d is a sum of at most k squares. The exact values of the Pythagoras number are known in very few cases. I will review some history and bounds on the Pythagoras number, and then explain how to establish Pythagoras number for ternary forms of degree 8 and 10. Based on joint work with Alex Dunbar, Rainer Sinn, Mauricio Velasco and Shixuan Zhang.

5. Mario Kummer (TU Dresden). Spectrahedral Shadows and Completely Positive Maps on Real Closed Fields.

Nemirovski asked in his plenary talk at ICM 2006 whether every convex semialgebraic set is a spectrahedral shadow. In a celebrated and surprising work, Scheiderer answered this question to the negative. In my talk I will present a joint work with Manuel Bodirsky and Andreas Thom in which we provide further counter-examples. We use results on completely positive maps on real closed fields and a simple model theoretic observation.