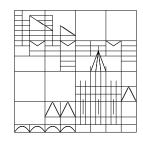
University of Konstanz Department of Mathematics and Statistics Prof. Dr. Robert Denk

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## Nonlinear partial differential equations Exercises and Questions

**Question 1.** Let E be a topological space.

a) Define a topology  $\tau$  on  $\mathbb{R}$  such that the space of all upper semicontinuous functions  $f: E \to \mathbb{R}$  coincides with the space of all continuous functions  $f: E \to (\mathbb{R}, \tau)$ .

b) Let  $f \in USC(E)$ . What can be said on the range of f?

c) Let  $\Lambda$  be a set and  $\{f_{\lambda} : \lambda \in \Lambda\} \subset USC(E)$ . Are the functions  $\sup_{\lambda \in \Lambda} f_{\lambda}$  and  $\inf_{\lambda \in \Lambda} f_{\lambda}$  semicontinuous? What can be said if  $\Lambda$  is finite?

**Question 2.** Let  $G \subset \mathbb{R}^n$  be a domain,  $u \in USC(G)$  and  $x_0 \in G$ . Show that  $J^+u(x_0)$  is the set of all  $(\nabla \varphi(x_0), \nabla^2 \varphi(x_0))$  with  $\varphi \in C^2(U)$  such that  $u - \varphi$  has a local maximum at  $x_0$ .

**Question 3.** Let  $G \subset \mathbb{R}^n$  be a domain and  $u: G \to [-\infty, \infty]$  be a function. Define

$$u^*(x) := \lim_{r \searrow 0} \left[ \sup\{u(y) : y \in G, |x - y| < r \} \right] \quad (x \in G).$$

Show that  $u^*$  is the smallest upper semicontinuous function (with values in  $[-\infty, \infty]$ ) greater or equal to u.

Please prepare your answers for presentation and discussion on May 6th (15.15-16.45, F 420).