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Department of Mathematics and Statistics
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Nonlinear partial differential equations

Exercises and Questions, Part 2

Question 1. Let $G \subset \mathbb{R}^n$ be a domain and $u \in USC(G)$. Show that the set of all points $x \in G$ with non-empty superjet $J^+u(x)$ is dense in G.

Hint: For $x \in G$ and $\varepsilon > 0$, define x_ε as the maximum point of the function $y \mapsto u(y) - \frac{1}{\varepsilon}|y-x|^2$ in a closed ball around x.

Question 2. Compute $J_U u(x)$ for all $x \in U$ in the following cases:

(a) $U = (-1, 1)$, $u(x) := 1 - |x|$ $(x \in U)$,

(b) $U = \mathbb{R}$, $u(x) := \chi_{[0,\infty)}(x)$ $(x \in U)$ (Heaviside-Funktion),

(c) $U = [0, 1]$, $u(x) := 0$ $(x \in U)$.

Question 3. Let $G \subset \mathbb{R}^n$ be a bounded domain, $f \in C(\overline{G})$, $A, B \subset \mathbb{R}^m$ be compact sets, $\sigma \in C(A \times B \times \overline{G}, \mathbb{R}^{n\times n})$, $(a, b, x) \mapsto \sigma^{(a,b)}(x)$ with

$$\|\sigma^{(a,b)}(x) - \sigma^{(a,b)}(y)\| \le L|x-y| \quad (x, y \in \overline{G},\, a \in A,\, b \in B)$$

for a constant $L > 0$. Define F by

$$F(x, r, p, X) := \sup_{a\in A} \inf_{b\in B} \big[-\operatorname{tr}(\sigma^{(a,b)}(x)(\sigma^{(a,b)}(x))^\top X) - f(x) \big]$$

for $x \in \overline{G}$, $r \in \mathbb{R}$, $p \in \mathbb{R}^n$, $X \in \mathbb{R}^{n\times n}_{\text{sym}}$ (Isaacs equation). Show that F satisfies condition (M2).

Please prepare your answers for presentation and discussion on May 20th (15.15-16.45, F 420).