University of Konstanz
Department of Mathematics and Statistics
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April 28, 2014

Nonlinear partial differential equations

Exercises and Questions

Question 1. Let E be a topological space.

a) Define a topology τ on $\mathbb{R}$ such that the space of all upper semicontinuous functions $f\colon E \to \mathbb{R}$ coincides with the space of all continuous functions $f\colon E \to (\mathbb{R}, \tau)$.

b) Let $f \in USC(E)$. What can be said on the range of f?

c) Let Λ be a set and $\{f_\lambda : \lambda \in \Lambda\} \subset USC(E)$. Are the functions $\sup_{\lambda\in\Lambda} f_\lambda$ and $\inf_{\lambda\in\Lambda} f_\lambda$ semicontinuous? What can be said if Λ is finite?

Question 2. Let $G \subset \mathbb{R}^n$ be a domain, $u \in USC(G)$ and $x_0 \in G$. Show that $J^+u(x_0)$ is the set of all $(\nabla\varphi(x_0), \nabla^2\varphi(x_0))$ with $\varphi \in C^2(U)$ such that $u - \varphi$ has a local maximum at x_0.

Question 3. Let $G \subset \mathbb{R}^n$ be a domain and $u\colon G \to [-\infty, \infty]$ be a function. Define

$$u^*(x) := \lim_{r \searrow 0} \big[\sup\{u(y) : y \in G, |x-y| < r\} \big] \quad (x \in G).$$

Show that u^* is the smallest upper semicontinuous function (with values in $[-\infty, \infty]$) greater or equal to u.

Please prepare your answers for presentation and discussion on May 6th (15.15-16.45, F 420).