Universität Konstanz

KWIM Conference 2023 Cross-Diffusion Systems: Analysis and Stochastics

Book of Abstracts

27th February – 3rd March 2023

The main objective of this conference is to bring together researchers working on quasilinear (S)PDEs with a main focus on cross-diffusion systems and their applications. Various aspects from the theory of cross-diffusion (S)PDEs such as mathematical modelling, dynamical systems and numerical analysis will be discussed.
The conference contains two parts. In the first half, experts working on these topics will provide lectures in order to give young researchers an insight into the theory of such (S)PDEs. A follow-up workshop with invited and contributed talks will take place in the second half (of the week).

Organisers: Jun. Prof. Alexandra Neamtu and Jun. Prof. Lara Trussardi, University of Konstanz

Department of Mathematics and Statistics

Lecturers

Laurent Desvillettes *Université Paris Cité, IMJ-PRG (France)*
Title: New estimates for cross diffusion systems coming out of biology and medicine
Abstract: (reaction -) cross diffusion systems are systems of parabolic equations in which the diffusion rate of a given quantity can depend upon the other quantities appearing in the system. When this dependence concerns only one of the unknowns, the system is sometimes called "triangular", and the question of existence of weak or strong solutions can be studied by techniques which are significantly different from the ones which are used in the general case. We propose to show on a few specific examples coming out of biology/medecine some of those techniques, and some of the current mathematical challenges.

Benjamin Fehrman *Oxford University (UK)*
Title: Well-posedness of the Dean–Kawasaki equation with correlated noise
Abstract: The purpose of these lectures will be to introduce a well-posedness theory for a general class of SPDEs including, in particular, the generalized Dean–Kawasaki equation and nonlinear Dawson–Watanabe equation with correlated noise. These equations arise in diverse contexts—such as, among others, non-equilibrium fluctuations of interacting particle systems, branching interacting diffusions, and stochastic mean-curvature flows—and their well-posedness is challenging due to nonlinear and potentially degenerate diffusions, fluctuations entering in the form of stochastic conservation laws, and irregular noise coefficients including the square root. The aim of the first lecture will be to derive the equation's kinetic formulation, which is based on and generalizes the notion of an entropy solution. The necessity of this formulation is due both to the degeneracy of the diffusion, where the equation is expected to behave like a scalar conservation law, and to the fact that certain fluxes will not be integrable, which will require a renormalized solution theory. In the second lecture, we will explain how the kinetic formulation developed in the first lecture can be used to treat the Dean–Kawasaki equation. In some ways the kinetic form linearizes the equation, and the nonlinear regularity of the solution is translated to properties of an associated parabolic defect measure. By understanding the fine properties of this measure, we will explain how to treat the singularities that arise from the singular noise coefficients, including the square root.

Benjamin Gess *MPI MiS, Leipzig, Bielefeld University (Germany)*
Title: Fluctuations in conservative systems and SPDEs
Abstract: Fluctuations are ubiquitous in non-equilibrium conservative systems. The analysis of their large deviations lead to macroscopic fluctuation theory (MFT), a general framework for non-equilibrium statistical mechanics. MFT is based on a constitutive formula for large fluctuations around thermodynamic variables, and can be justified from fluctuating hydrodynamics. The latter postulates conservative, sin-

gular SPDEs to describe fluctuations in systems out of equilibrium. Both theories are informally linked via zero noise large deviations principles for SPDEs. In this talk, we will introduce several examples demonstrating these relations, and leading to a class of conservative SPDEs. The analysis of large deviations of interacting particle systems and conservative SPDEs will then lead to intricate and open problems for PDEs with irregular coefficients. In the last part of the talk, we present one positive result in this direction, solving a long-standing open problem in the proof of a full large deviation result for the zero-range process.

Erika Hausenblas *Montanuniversität Leoben (Austria)*
Title: Existence of a martingale solution of a stochastic chemotaxis system with underlying fluid dynamic
Abstract: The talk will be about a recent work, where we applied a kind of Schauder-Tychonov Theorem to get existence of a martingale solution of some coupled stochastic equations. In the first step, we will introduce the setting of the problem and the necessary definition and concepts.
Firstly, we will present a kind of Meta Theorem to show the existence of a martingale solution for coupled systems of non-linear stochastic differential equations. The idea is first to linearise the system by rewriting the non-linear part in a linear part acting on a given process ξ. This is done in such a way that the fixpoint with respect to ξ would be the solution. In the next step, we show that such a fixed point exists by showing compactness and continuity on an invariant subset of the mapping.
Secondly, to illustrate applicability of the Meta Theorem, we analyse the existence of a martingale solution of a highly nonlinear chemotaxis system with underlying fluid dynamic. In particular, we investigate the coupled system

$$\begin{cases} dn+\delta_n\mathbf{u}\cdot\nabla\mathbf{n} &= [r_n\Delta|n|^{q-1}n-\chi\text{div}(n\nabla c)]\;dt+g_{\gamma_1}(n)\circ dW_1,\\ dc+\delta_c\mathbf{u}\cdot\nabla\mathbf{c}+\mathbf{f}(\mathbf{c}) &= [r_c\Delta c-\alpha c+\beta n]\;dt+g_{\gamma_2}(c)\circ dW_2,\\ d\mathbf{u} &+ [\mathbf{u}\cdot\nabla\mathbf{u}+\nabla\mathbf{P}-\mathbf{\Delta u}]\,dt=n\nabla\Phi dt+\sigma_{\gamma_3}\,dW_3,\\ \nabla\cdot\mathbf{u} &= 0\end{cases}$$

with initial condition $(n_0,c_0,\mathbf{u}_0)$ on a filtered probability space $\mathfrak{A}$ and W_1, W_2, and W_3 be three independent time homogeneous spatial Wiener processes over $\mathfrak{A}$. Here n is the cell density, c is the concentration of the chemical signal, and $\mathbf{u}$ is a vector field over $\mathcal{O}$. The positive terms r_n and r_c are the diffusivity of the cells and chemoattractant, respectively, the positive value χ is the chemotactic sensitivity, $\alpha\geq 0$ is the so-called damping constant, f models the proliferation.
It is a joint work with Michael Hoegele, and Boris Jidjou Moghomye.

Ansgar Jüngel *TU Wien (Austria)*
Title: Cross-diffusion systems with entropy structure
Abstract: Cross-diffusion equations describe the diffusive interaction in multicomponent systems like in population dynamics, biological cells, and gas mixtures. The models consist of strongly coupled parabolic equations with a full diffusion matrix.

The major challenge is that the diffusion matrix is generally neither symmetric nor positive definite in applications. A common feature of the models is that they possess a formal gradient-flow or entropy structure. This structure provides gradient estimates and is exploited to analyze the cross-diffusion systems mathematically. In this mini-course, the global existence analysis of such systems is presented (boundedness-by-entropy method) and applied to some examples from biology and physics. An outlook to further topics like regularity of solutions, large-time asymptotics, and weak-strong uniqueness will be also given.

Christian Kuehn *TU Münich (Germany)*
Title: Cross-Diffusion Dynamics: From Analysis to Numerics
Abstract: In my talk, I will first explain the approach provided by dynamical systems theory to understand reaction-diffusion systems. Then, I will proceed to explain the additional difficulties posed by cross-diffusion dynamics and indicate several analytical results. Next, I will proceed to explain, why for global results we need additional computational tools such as numerical continuation. Finally, I will demonstrate, how these methods also extend to stochastic cross-diffusion problems.

Speakers

Maxime Breden *École Polytechnique, Paris (France)*
Title: Rigorous computation and continuation of steady states for cross-diffusion systems in population dynamics
Abstract: Cross-diffusion models are notorious for producing many different patterns, which makes such systems both very interesting and challenging to study mathematically. Using mostly the SKT system as an example, I will explain how numerical simulations can be combined with a priori and a posteriori estimates in order to obtain very flexible computer-assisted proofs of existence of steady states. I will also showcase that these techniques can be adapted in a straightforward way to produce rigorous continuation results, allowing to prove the existence of branches of solutions, and to rule out bifurcations.

Clément Cancés *Inria Lille - Nord Europe (France)*
Title: Finite volumes for dissipative mixtures with non-local mild volume filling
Abstract: Multicomponent diffusion in incompressible mixtures involves pressure forces originating from the incompressibility constraint. While volume filling is often incorporated in models in a local (or strong) way prescribing the total volume flux to vanish, we rather consider here a relaxed version where the total volume flux is only required to be divergence free, leading to a milder constraint but non-local

pressure field. We will more particularly discuss the construction of entropy stable finite volume schemes for such problems. This is a joint work with Antoine Zurek.

Li Chen *University of Mannheim (Germany)*
Title: Global weak solutions to the compressible Cucker-Smale-Navier-Stockes system in a bounded domain
Abstract: A coupled kinetic-fluid model is investigated, which describes the dynamic behavior of an ensemble of Cucker-Smale flocking particles interacting with a viscous fluid in a three-dimensional bounded domain. This system consists of a kinetic Cucker-Smale equation and a compressible Navier-Stokes system with nonhomogeneous boundary conditions. The global existence of weak solutions to this system with adiabatic coefficient $\gamma > 3/2$ is established. This is a joint work with Yue Li and Nicola Zamponi.

Federico Cornalba *IST (Austria)*
Title: Space-time multilevel Monte Carlo discretisation of Dean-Kawasaki models
Abstract: the Dean—Kawasaki model — a highly singular stochastic PDE — is a basic equation of Fluctuating Hydrodynamics. This model has been shown to be a statistically equivalent representation for the empirical density of underlying interacting particle systems, and is being used more and more for quantitative estimation of fluctuations in such systems. We consider a space-time Multilevel Monte Carlo scheme (MLMC) based on the finite-difference discrete Dean—Kawasaki model provided in [F. Cornalba and J. Fischer, arXiv:2109.06500]. We show that, subject to the standard Courant-Friedrichs-Lewy (CFL) condition $\tau/h^2 < 1$ and subject to the average particle number per grid cell being of the order $h^{-\alpha}$ ($\alpha \geq 0$), our MLMC scheme entails a computational speed-up factor of $h^{-\min\{\alpha;d\}}$ over the standard Monte Carlo method for all space dimensions $d \leq 4$. This means that our MLMC method starts gaining efficiency as soon as there are more particles than grid points. Variants of the same result are also discussed when the CFL condition is violated. Based on joint work in progress with Julian Fischer (ISTA) and Quinn Winters (TU Munich).

Lucilla Corrias *Université d'Evry Val d'Essonne (France)*
Title: Fast reaction and cross diffusion: models inspired by dietary diversity
Abstract: In this talk, we will consider a class of cross diffusion systems obtained as the equilibrium limit of reaction diffusion systems with fast reaction terms. The fast reaction diffusion systems model the dynamic of two species, one of which switch between two different states. We will show how the limit, as the conversion rate goes to zero, is rigorously obtained through a family of energy functionals. The existence of weak solutions of the corresponding cross diffusion systems is thus obtained. A weak-strong stability/uniqueness result will be also presented. This is a joint work with E. Brocchieri and L. Desvillettes.

Julian Fischer *IST (Austria)*

Title: A rigorous approach to the Dean-Kawasaki equation of fluctuating hydrodynamics

Abstract: Fluctuating hydrodynamics provides a framework for approximating density fluctuations in interacting particle systems by suitable SPDEs. The Dean-Kawasaki equation - a strongly singular SPDE - is perhaps the most basic equation of fluctuating hydrodynamics; it has been proposed in the physics literature to describe the fluctuations of the density of N diffusing weakly interacting particles in the regime of large particle numbers N. The strongly singular nature of the Dean-Kawasaki equation presents a substantial challenge for both its analysis and its rigorous mathematical justification: Besides being non-renormalizable by approaches like regularity structures, it has recently been shown to not even admit nontrivial martingale solutions.

In this talk, we give an overview of recent quantitative results for the justification of fluctuating hydrodynamics models. In particular, we give an interpretation of the Dean-Kawasaki equation as a "recipe" for accurate and efficient numerical simulations of the density fluctuations for weakly interacting diffusing particles, allowing for an error that is of arbitarily high order in the inverse particle number. Based on joint works with Federico Cornalba, Jonas Ingmanns, and Claudia Raithel.

Manuel Gnann *TU Delft (the Netherlands)*

Titel: Non-negative Martingale Solutions to the Stochastic Thin-Film Equation with Nonlinear Gradient Noise

Abstract: We prove the existence of non-negative martingale solutions to a class of stochastic degenerate-parabolic fourth-order PDEs arising in surface-tension driven thin-film flow influenced by thermal noise. The construction applies to a range of mobilites including the cubic one which occurs under the assumption of a no-slip condition at the liquid-solid interface. Since their introduction more than 15 years ago, by Davidovitch, Moro, and Stone and by Grün, Mecke, and Rauscher, the existence of solutions to stochastic thin-film equations for cubic mobilities has been an open problem, even in the case of sufficiently regular noise. Our proof of global-in-time solutions relies on a careful combination of entropy and energy estimates in conjunction with a tailor-made approximation procedure to control the formation of shocks caused by the nonlinear stochastic scalar conservation law structure of the noise. The talk is based on joint work with Konstantinos Dareiotis (University of Leeds), Benjamin Gess (Bielefeld University/MPI Leipzig), and Günther Grün (University of Erlangen-Nuremberg).

Antoine Hocquet *TU Berlin (Germany)*

Title: McKean–Vlasov equations with rough common noise and quenched propagation of chaos

Abstract: I will show well-posedness and propagation of chaos for McKean–Vlasov equations with rough common noise and progressively measurable coefficients. The

results are valid under minimal regularity assumptions on the coefficients, in agreement with the respective requirements of Itô and rough path theory. To achieve these goals, I will introduce the framework of rough stochastic differential equations recently developed by K. Lê, P. Friz and myself.

Florian Huber *University of Vienna (Austria)*

Title: Stochastic cross-diffusion systems

Abstract: We consider a general class of cross diffusion systems, satisfying entropy structure conditions, perturbed by a random process. The classical entropy approach turned out not to be well suited in this setting. Hence we present a novel approach using a transformation technique which combines the entropy strucure of the equation as well as a regularization procedure to obtain probabilistically weak solutions for the proposed stochastic partial differential equations. Since our approach does not rely on the structure of the randomness, it also presents a new approach for deterministic equations with an entropy structure. Lastly, we want to give a short outlook on the long-time behaviour of these SPDEs. Under certain conditions,(generalized) Feller theory can be used to obtain existence of invariant measures.

Lisa Maria Kreusser *University of Bath (UK)*

Title: On anisotropic diffusion equations for label propagation

Abstract: In many problems in data classification, it is desirable to assign labels to points in a point cloud where a certain number of them is already correctly labeled. In this talk, we propose a microscopic ODE approach, in which information about correct labels propagates to neighboring points. Its dynamics are based on alignment mechanisms, often used in collective and consensus models. We derive the respective continuum description, which corresponds to an anisotropic diffusion equation with a reaction term. Solutions of the continuum model inherit interesting properties of the underlying point cloud. We discuss the qualitative behavior of solutions and exemplify the results with micro- and macroscopic simulations. This is joint work with Marie-Therese Wolfram.

Thomas Lepoutre *Lyon (France)*
Title: Improved duality estimates: the time discrete case
Abstract: We adapt the improved duality estimates for bounded coefficients derived by Canizo et al. to the framework of cross diffusion. We derive a time discrete version of their results and apply it to an implicit semi-discretization in time of the cross diffusion systems. This can be applied in particular to cross diffusion systems with bounded cross diffusion pressures.

Sara Merino-Aceituno *University of Vienna (Austria)*
Title: Questions stemming from models for collective dynamics
Abstract: In this talk I will review some questions that arise around the classical Vicsek model - which is a model for collective dynamics where agents move at a constant speed while trying to adopt the averaged orientation of their neighbors, up to some noise. I will discuss ideas like the derivation of the macroscopic equations, phase transitions, effects of the environment, and generalizations of the model.

Maria Neuss-Radu *University of Erlangen-Nürnberg (Germany)*
Title: Effective models for nonlinear drift-diffusion in porous media
Abstract: We consider a nonlinear drift-diffusion system for multiple charged species in a porous medium in 2D and 3D with periodic microstructure. The system consists of a transport equation for the concentration of the species and Poisson's equation for the electric potential. The diffusion terms depend nonlinearly on the concentrations. We consider non-homogeneous Neumann boundary condition for the electric potential. The aim is the rigorous derivation of an effective (homogenized) model in the limit when the scale parameter ϵ tends to zero. This is based on uniform *a priori* estimates for the solutions of the microscopic model. The crucial result is the uniform L^∞-estimate for the concentration in space and time. This result exploits the fact that the system admits a nonnegative energy functional which decreases in time along the solutions of the system. By using weak and strong (two-scale) convergence properties of the microscopic solutions, effective models are derived in the limit $\epsilon \to 0$ for different scalings of the microscopic model. This is a joint work with Apratim Bhattacharya (Erlangen) and Markus Gahn (Heidelberg).

Mariya Ptashnyk *Heriot-Watt University, Edinburgh (Scotland)*
Title: Multiscale analysis for cross-diffusion systems
Abstract: Two aspects of multiscale analysis of parabolic problems will be considered: periodic stochastic homogenization and derivation of macroscopic equations from the interacting particles systems. We will consider cross-diffusion systems defined in heterogeneous or perforated domains, with periodic and random microstructures. Applying techniques of periodic and stochastic homogenization, macroscopic (effective) cross-diffusion equations will be derived from the microscopic models defined on the scale of the microstructure. In the context of the derivation of limiting equations from the particles systems, we will present the main ideas on the derivati-

on of a fractional cross-diffusion system from a multi-species system of moderately interacting particles.

Jonas Sauer *FSU Jena (Germany)*
Title: Optimal Sobolev Regularity for a Nonlinear Fractional Heat Equation of Porous Media Type
Abstract: We consider solutions to the nonlocal, nonlinear, degenerate equation

$$\begin{cases} \partial_t u + (-\Delta)^\alpha u^{[m]} = S & \text{in } (0,T) \times \mathbb{R}^d, \\ u(0) = u_0 & \text{in } \mathbb{R}^d \end{cases}$$

where $\alpha \in (0,1)$, $S \in L^1(0,T;L^1(\mathbb{R}^d))$, $u_0 \in L^1(\mathbb{R}^d)$ and $u^{[m]} := |u|^{m-1}u$ with $m > 1$. This is a nonlocal variant of the porous medium equation (which corresponds to $\alpha = 1$), for which optimal space-time regularity has been established recently by B. Gess, the speaker and E. Tadmor (B. Gess, J. Sauer and E. Tadmor. Optimal Regularity in Time and Space for the Porous Medium Equation, Anal. PDE 13(8):2441–2480, 2020).
In this talk I explain how the method of kinetic formulation and averaging lemmas can be utilized to obtain (both in the local and nonlocal case) Sobolev regularity results that are in line with the optimal regularity suggested by scaling arguments and which are consistent with the limiting linear case $m = 1$.
The talk is based on a joint work with B. Gess and E. Tadmor (2020) and on ongoing work with B. Gess and L. Diening.

Stefanie Sonner *Radboud University Nijmegen (The Netherlands)*
Title: Travelling waves for degenerate reaction diffusion systems modelling cellulolytic biofilms
Abstract: Biofilms are dense aggregations of bacterial cells attached to a surface and held together by a self-produced matrix of extracellular polymeric substances. They affect many aspects of human life and play a crucial role in natural, medical and industrial settings. We consider continuum models for spatially heterogeneous biofilm communities formulated as quasilinear reaction diffusion systems. Their characteristic and challenging feature is the two-fold degenerate diffusion coefficient for the biomass density comprising a polynomial degeneracy (as known from the porous medium equation) and a fast diffusion singularity as the biomass density approaches its maximum value.
In this talk we focus on a recent model for cellulolytic biofilms that play an important role in the production of cellulosic ethanol. Different from traditional biofilm models where the biofilm colonies grow into the aqueous phase and nutrients are transported by diffusion, bacteria colonize, consume and degrade a cellulosic substratum that supports them. Hence, the nutrients are immobilized and modeled by an ordinary differential equation. We show results on the well-posedness of the model and prove the existence of traveling wave solutions. Invading fronts had earlier been observed in biological experiments on cellulolytic biofilms as well as in numerical simulations of the model.

Cinzia Soresina *University of Graz (Austria)*
Title: Cross-diffusion systems in population dynamics: derivation, bifurcations and pattern formation
Abstract: In population dynamics, cross-diffusion describes the influence of one species on the diffusion of another. The cross-diffusion SKT model was proposed to account for stable inhomogeneous steady states exhibiting spatial segregation of two competing species. Even though the reaction part does not present the activator-inhibitor structure, the cross-diffusion terms are the key ingredient for the appearance of spatial patterns. We provide a deeper understanding of the conditions required for non-homogeneous steady states to exist, focusing on multistability regions and on the presence of time-periodic spatial patterns, by combining a detailed linearised and weakly non-linear analysis with advanced numerical bifurcation methods via the continuation software pde2path (M. Breden, C. Kuehn, C. Soresina, On the influence of cross-diffusion in pattern formation, Journal of Computational Dynamics, 8(2):21, 2021. C. Soresina, Hopf bifurcations in the full SKT model and where to find them, Discrete and Continuous Dynamical Systems - S, 15(9):2673-2693, 2022).
Even though the particular form of cross-diffusion terms in the SKT model may seem artificial, they naturally incorporates processes occurring at different time scales. It can be easily seen, at least at a formal level, that cross-diffusion appears in the fast-reaction limit of a "microscopic"model (in terms of time scales) presenting only standard diffusion and fast-reaction terms.mThe same approach can also be exploited in other contexts,e.g. predator-prey interactions, plant ecology and epidemiology.

Jan-Eric Sulzbach *TU Munich (Germany)*
Title: Generalization of Fenichel's theorem for fast-reaction PDEs
Abstract: Fast-slow dynamical systems in finite dimension have been studied by many authors over the last decades. However, it is very challenging to transfer the results to infinite dimensions. In this talk we focus on fast-reaction systems in general Banach spaces and we extend the classical Fenichel theory to this infinite-dimensional setting. Using insights from the finite-dimensional case, we first transfer the notion of normal-hyperbolicity. Then, we show that the solution of the fast-reaction system can be approximated by the corresponding slow flow of the limit system. Introducing an additional parameter that stems from a splitting in the slow variable space, we construct a family of slow manifolds and prove that the slow manifolds are close to the critical manifold.

Christina Surulescu *TU Kaiserslautern (Germany)*
Title: A nonlocal perspective on tactic cell migration
Abstract: Increasing experimental evidence suggests that cells are able to sense physical and chemical cues in their environment, up to several cell diameters around their current position. This motivated the development of continuous mathematical models which account for various types of (spatial) nonlocalities, most of them addressing cell-cell and/or cell-matrix adhesions. They can be classified according, e.g., to the type of featured nonlocality.

We present a modeling approach with spatial nonlocality in the drift described via integral operators applied directly to gradients of signal-dependent quantities and being able to handle hapto- and chemotactic behavior in a unified manner. For a shrinking radius of the cell sensing region, the nonlocal formulation converges to a local reaction-diffusion-taxis setting. Numerical simulations in 1D are used to illustrate the results and compare to previous models.
This is joint work with Maria Eckardt (RPTU Kaiserslautern-Landau), Kevin Painter (Politecnico di Torino), and Anna Zhigun (Queen‘s Univ. Belfast).

Báo Quôc Tăng *University of Graz (Austria)*
Title: Rigorous derivation of the Michaelis-Menten kinetic in the presence of diffusion for enzyme reactions
Abstract: Michaelis-Menten kinetic is one of the most used when modelling enzyme-, or more generally catalytic-, reactions. In the case of homogeneous medium, i.e. the (bio-)chemical concentrations depend solely on time, both formal and rigorous derivations of MM from mass action kinetic have been studied extensively and thoroughly in the last decades. For heterogeneous medium, the modelling should take into account the diffusion of substances, which leads to a system of partial differential equations. In this case, interestingly, only formal derivation of MM from mass action kinetic has been investigated. In this talk, we present, up to our knowledge, the first rigorous derivation of MM in the presence of diffusion. The proof utilises an improved duality technique and a modified energy method. This is based on a joint work with Bao-Ngoc Tran (University of Graz).

Jonas Tölle *Aalto University (Finnland)*
Title: Singular limits for stochastic equations
Abstract: We study singular limits of nonlinear stochastic evolution equations in the interplay of disappearing strength of the noise and increasing roughness of the noise, so that the noise in the limit would be too rough to define a solution to the limiting equations. Simultaneously, the limit is singular in the sense that the leading order differential operator may vanish. Although the noise is disappearing in the limit, additional deterministic terms appear due to renormalization effects. We give an abstract framework for the main error estimates, that first reduce to bounds on a residual and in a second step to bounds on the stochastic convolution. Moreover, we apply it to a singularly regularized Allen-Cahn equation and the Cahn-Hilliard/Allen-Cahn homotopy. Possible applications of our method to nonlinear cross-diffusion systems will be discussed. See $https://arxiv.org/abs/2204.09545$
Joint work with Dirk Blömker, University of Augsburg, Germany.

Ariane Trescases *Institut de Mathématiques de Toulouse (France)*
Title: Models for chemotaxis with local sensing
Abstract: We present a class of triangular cross-diffusion systems that model cell chemotaxis with local sensing, that is, the cells respond to a certain concentrati-

on of chemoattractant perceived locally (as opposed to gradient sensing, when the cells are able to perceive a gradient of concentration). We study the well-posedness and long-time behaviour of these models, using entropy and duality methods.

Havva Yoldas *TU Delft (the Netherlands)*
Title: A variational approach for the existence of solutions of a degenerate cross-diffusion model
Abstract: In this talk, we look at a cross-diffusion system consisting of two Fokker-Planck equations where the gradient of the density for each species acts as a potential for the other one. The system is the gradient flow for the Wasserstein distance of a functional which is not lower semi-continuous, and the system is not well-posed. We compute the convexification of the integral and provide an existence proof in a suitable sense for the gradient flow of the corresponding relaxed functional. The talk is based on a joint work with R. Ducasse and F. Santambrogio (Calc. Var. PDE, 2023).

Nicola Zamponi *TU Wien (Austria)*
Title: Analysis of a multispecies Fokker-Planck system
Abstract: A multispecies Fokker-Planck system is studied, which is derived from a spatially homogeneous multispecies Vlasov-Landau equation by linearizing the binary collision operator around the global Maxwellian equilibrium. The resulting equations conserve the mass of each component as well as total momentum and total energy, and dissipate the (mathematical) entropy. The multispecies Fokker-Planck equations constitute a cross-diffusion system where the cross-diffusion comes from coefficients appearing in the equations which depend nonlocally on the solution via its moments. We prove global existence of weak solutions to the system fulfilling the aforementioned conservation properties and entropy decay. The existence proof relies on an approximated system on a generic ball in the three-dimensional Euclidean space involving a p-Laplacian regularization term and truncated moments. Uniform estimates for the approximated solution are deduced from energy and entropy balance together with a lower bound for the temperature. Compactness for the approximated solution results from Aubin-Lions Lemma coupled with a Sobolev's embedding involving the whole three-dimensional Euclidean space.

Antoine Zurek *Université de Technologie de Compiègne (France)*
Title: Analysis of a numerical scheme for a nonlocal cross-diffusion system
Abstract: In this talk, I will consider a nonlocal version of the Shigesada-Kawazaki-Teramoto (SKT) cross-diffusion system, which arises in population dynamics. This system has entropy dissipation properties on which one can rely to design a robust and convergent numerical scheme for its numerical simulation. In terms of numerical analysis, I will present discrete compactness techniques, entropy-dissipation estimates and a new adaptation of the so-called duality estimates for parabolic equations in Laplacian form. I will also present numerical experiments illustrating

the influence of the nonlocality in the system: on convergence properties, as an approximation of the local system and on the development of diffusive instabilities. This is a joint work with Maxime Herda (Inria).