Positive polynomials and moment problems

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Abstract: Hilbert's 17th problem asked whether a real polynomial $p(x_1,..,x_n)$ which takes non-negative values as a function on R^n is a finite sum of squares (SOS) of real rational functions $q(x_1,..,x_n)/r(x_1,..,x_n)$. A complete positive answer was provided by Artin and Schreier (1927), giving birth to real algebraic geometry. The question when the (SOS) representation is denominator free is however of particular interest for applications. In his pioneering 1888 paper, Hilbert gave a general answer (in terms of degree and number of variables). Subsequent general results, such as Krivine's Positivstellensatz, pertain to a relative situation, where one considers polynomials non-negative on a basic closed semi-algebraic set K and SOSs weighted with inequalities defining K. Stronger results hold when K is compact; the Archimedean Positivstellensatz of Putinar and Jacobi-Prestel is a fundamental tool in theory and applications. By the classical Riesz-Haviland theorem (1930s), the problem of characterizing positive polynomials on a given closed subset K of R^n is dual to the finite dimensional moment problem (i.e. that of representing a linear functional on the polynomial algebra $R[x_1,..,x_n]$ as integration with respect to a Borel measure). An algebraic approach was taken in a series of papers by Ghasemi-Kuhlmann-Marshall (2013-2016) who study the moment problem on a general not necessarily finitely generated commutative unital real algebra, a context adapted to infinite dimensional moment problems. In this talk I will survey (with examples) various Positivstellensaetze and their corresponding moment problem interpretations.

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