Automorphisms of valued Hahn groups

joint work with Salma Kuhlmann

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- Let $(\Gamma, <)$ be a chain, $[\Gamma; \{A_\gamma : \gamma \in \Gamma\}]$ an ordered system of abelian groups.
- For an element $a = (a_\gamma)_{\gamma \in \Gamma} \in \prod_{\gamma \in \Gamma} A_\gamma$ we use the formal notation

$$a = \sum_{\gamma \in \Gamma} a_\gamma \mathbb{1}_\gamma$$

- The support of a is the set $\mathrm{supp}(a) = \{\gamma \in \Gamma : a_\gamma \neq 0\}$
- Denote by

$$\mathbb{G} := \mathop{\mathbf{H}}_{\gamma \in \Gamma} A_\gamma := \left\{ a = \sum_{\gamma \in \Gamma} a_\gamma \mathbb{1}_\gamma : \mathrm{supp}(a) \text{ is well ordered} \right\}$$

$$\coprod_{\gamma \in \Gamma} A_\gamma := \{a \in \mathbb{G} : \mathrm{supp}(a) \text{ is finite}\}$$

- for $a = \sum_{\gamma\in\Gamma} a_\gamma \mathbb{1}_\gamma,\ b = \sum_{\gamma\in\Gamma} b_\gamma \mathbb{1}_\gamma \in \mathbb{G}$ the componentwise addition

$$a + b := \sum_{\gamma\in\Gamma} (a_\gamma + b_\gamma)\mathbb{1}_\gamma$$

  makes $\mathbb{G}$ into a group.

Definition (Hahn group)

Let $S = [\Gamma; \{A_\gamma : \gamma \in \Gamma\}]$ be an ordered system of abelian groups. A Hahn group over S is a group G such that

$$\coprod_{\gamma\in\Gamma} A_\gamma \leq G \leq \mathbf{H}_{\gamma\in\Gamma} A_\gamma$$

$S = S(G)$ is called the skeleton of G;
$\mathbb{G}$ and $\coprod_{\gamma\in\Gamma} A_\gamma$ are called the maximal and minimal Hahn group over S.

Valuation, ordering and automorphisms

$v\colon G \to \Gamma \cup \{\infty\}$

$v(a) = \begin{cases} \min \operatorname{supp}(a) & a \neq 0 \\ \infty & a = 0 \end{cases}$

Valuation preserving automorphisms

$$v\text{-}\operatorname{Aut} G = \left\{ \begin{matrix} \sigma \in \operatorname{Aut} G : \forall a, b \in G \\ v(a) = v(b) \Rightarrow v(\sigma(a)) = v(\sigma(b)) \end{matrix} \right\}$$

Order preserving automorphisms. If all the A_γ are odered groups, we can order G lexicographically and we will denote by

$$o\text{-}\operatorname{Aut} G = \{\sigma \in \operatorname{Aut} G : \forall a, b \in G,\ a < b \Rightarrow \sigma(a) < \sigma(b)\}$$

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Theorem (Hahn, 1907)

Every ordered abelian group is isomorphic to a suitable Hahn group

Parallel with Hahn fields

- Let $(\Gamma, <)$ be a chain, $[\Gamma; \{A_\gamma : \gamma \in \Gamma\}]$ an ordered system of abelian groups.
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$$\mathbb{G} := \mathbf{H}_{\gamma\in\Gamma} A_\gamma := \left\{ a = \sum_{\gamma\in\Gamma} a_\gamma \mathbb{1}_\gamma : \begin{array}{c}\mathrm{supp}(a)\\ \text{is w.o.}\end{array} \right\}$$

- $a = \sum_{\gamma\in\Gamma} a_\gamma \mathbb{1}_\gamma,\ b = \sum_{\gamma\in\Gamma} b_\gamma \mathbb{1}_\gamma$

$$a + b := \sum_{\gamma\in\Gamma} (a_\gamma + b_\gamma)\mathbb{1}_\gamma$$

- $\mathbf{H}_{g\in G}\, k \simeq (\mathbb{K}, +)$

- Let $(G, +, 0, <)$ be a totally ordered abelian group and k a field.
- Denote by

$$\mathbb{K} := k((G)) := \left\{ a = \sum_{g\in G} a_g t^g : \begin{array}{c}\mathrm{supp}(a)\\ \text{is w.o.}\end{array} \right\}$$

- $a = \sum_{g\in G} a_g t^g,\ b = \sum_{g\in G} b_g t^g$

$$a + b := \sum_{g\in G} (a_g + b_g)t^g$$

$$ab = \sum_{g\in G} c_g t^g, \quad c_g = \sum_{r+s=g} a_r b_s$$

Parallel with Hahn fields

- $\coprod_{\gamma\in\Gamma} A_\gamma := \{a \in \mathbb{G} : \operatorname{supp}(a)$ is finite$\}$;

Definition (Hahn group)

A *Hahn group* is a group G such that

$$\coprod_{\gamma\in\Gamma} A_\gamma \subseteq G \subseteq \mathbb{G}$$

- $k[G] := \{a \in \mathbb{K} : \operatorname{supp}(a)$ is finite$\}$;
- $k(G) := \operatorname{Frac} k[G]$

Definition (Hahn field)

A *Hahn field* is a field K such that

$$k(G) \subseteq K \subseteq \mathbb{K}$$

Parallel with Hahn fields

- $v\colon G \to \Gamma \cup \{\infty\}$
  $$v(a) = \begin{cases} \min \operatorname{supp}(a) & a \neq 0 \\ \infty & a = 0 \end{cases}$$
- Skeleton: $S(G) = [\Gamma; \{A_\gamma : \gamma \in \Gamma\}]$

Theorem (Hahn 1907)

Every ordered abelian group is isomorphic to a suitable Hahn group.

- $v\colon K \to G \cup \{\infty\}$
  $$v(a) = \begin{cases} \min \operatorname{supp}(a) & a \neq 0 \\ \infty & a = 0 \end{cases}$$
- Valuation ring:
  $R_K = \{a \in K : v(a) \geq 0\}$
- Valuation ideal:
  $I_K = \{a \in K : v(a) > 0\}$
- Residue field: $\bar{K} = R_K / I_K \simeq k$

Theorem (Kaplansky 1942)

Every valued field is isomorphic to a suitable Hahn field

Automorphism groups

We fix a Hahn group G with skeleton $S(G) = [\Gamma; \{A_\gamma : \gamma \in \Gamma\}]$.

Definition

An automorphism τ of $S(G)$ consists of

- an order preserving bijection $\tau_\Gamma \colon \Gamma \to \Gamma$
- for all $\gamma \in \Gamma$ an isomorphism $\tau_\gamma \colon A_\gamma \to A_{\tau_\Gamma(\gamma)}$

We denote the group of automorphisms of $S(G)$ with composition by

$$\operatorname{Aut} S(G)$$

and write $\tau = [\tau_\Gamma; \{\tau_\gamma : \gamma \in \Gamma\}]$

Automorphism groups

We study the group

$$v\text{-}\operatorname{Aut} G = \left\{ \begin{matrix} \sigma \in \operatorname{Aut} G : \forall a, b \in G \\ v(a) = v(b) \Rightarrow v(\sigma(a)) = v(\sigma(b)) \end{matrix} \right\}$$

of valuation preserving automorphisms of G.

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of valuation preserving automorphisms of G.
Let $\sigma \in v\text{-}\operatorname{Aut} G$. To it we associate the automorphism $\bar{\sigma} = [\sigma_\Gamma; \{\sigma_\gamma : \gamma \in \Gamma\}] \in \operatorname{Aut} S(G)$ given by

$$\begin{aligned} \sigma_\Gamma &: \Gamma \to \Gamma; & v(a) &\mapsto v(\sigma(a)) \\ \sigma_\gamma &: A_\gamma \to A_{\sigma_\Gamma(\gamma)}; & a_\gamma &\mapsto \sigma(a_\gamma \mathbb{1}_\gamma)_{\sigma_\Gamma(\gamma)} \end{aligned}$$

Internal automorphisms

Obtain a group homomorphism

$$\begin{array}{rcl} \Phi_G\colon v\text{-}\operatorname{Aut} G & \longrightarrow & \operatorname{Aut} S(G) \\ \sigma & \longmapsto & [\sigma_\Gamma; \{\sigma_\gamma : \gamma \in \Gamma\}] \end{array}$$

Definition (Internal automorphisms)

$\operatorname{Int}\operatorname{Aut} G := \ker \Phi_G$

Internal automorphisms

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Definition (Internal automorphisms)

$\operatorname{Int}\operatorname{Aut} G := \ker \Phi_G$

- $\operatorname{Int}\operatorname{Aut} G \trianglelefteq v\text{-}\operatorname{Aut} G$
- $\sigma \in \operatorname{Int}\operatorname{Aut} G \Rightarrow \forall a \quad v(a) = v(\sigma(a))$ (valuation fixing)

Lifting property

Let $\tau = [\tau_\Gamma; \{\tau_\gamma : \gamma \in \Gamma\}] \in \operatorname{Aut} S(G)$.
Then τ induces an automorphism $\tilde{\tau} \in v\text{-}\operatorname{Aut} \mathbb{G}$ defined by

$$\tilde{\tau}\left(\sum_{\gamma\in\Gamma} a_\gamma \mathbb{1}_\gamma\right) = \sum_{\gamma\in\Gamma} \tau_\gamma(a_\gamma)\mathbb{1}_{\tau_\Gamma(\gamma)}$$

Definition

G has the canonical lifting property if, for all $\tau \in \operatorname{Aut} S(G)$ we have $\tilde{\tau}|_G \in \operatorname{Aut} G$.
In other words, if the map

$$\begin{array}{rccc} \Psi_G : & \operatorname{Aut} S(G) & \to & v\text{-}\operatorname{Aut} G \\ & [\tau_\Gamma; \{\tau_\gamma : \gamma \in \Gamma\}] & \mapsto & \tilde{\tau}|_G \end{array}$$

is a section of Φ_G.

External automorphisms

Definition (External automorphisms)

Let G have the canonical lifting property. Then

$$\operatorname{Ext}\operatorname{Aut} G := \operatorname{im} \Psi_G$$

is the group of *external automorphisms.*

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Let G have the canonical lifting property. Then

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is the group of *external automorphisms.* So

$$\operatorname{Ext}\operatorname{Aut} G \simeq \operatorname{Aut} S(G)$$

$$\operatorname{Int}\operatorname{Aut} G \hookrightarrow v\text{-}\operatorname{Aut} G \underset{\Psi_S}{\overset{\Phi_S}{\rightleftarrows}} \operatorname{Aut} S(G)$$

Decomposition Theorem

Theorem 1

Let G have the lifting property. Then

$$\begin{aligned} v\text{-}\operatorname{Aut} G &= \operatorname{Int}\operatorname{Aut} G \rtimes \operatorname{Ext}\operatorname{Aut} G \\ &\simeq \operatorname{Int}\operatorname{Aut} G \rtimes \operatorname{Aut} S(G) \end{aligned}$$

Decomposition Theorem

Theorem 1

Let G have the lifting property. Then

$$\begin{aligned} v\text{-}\operatorname{Aut} G &= \operatorname{Int}\operatorname{Aut} G \rtimes \operatorname{Ext}\operatorname{Aut} G \\ &\simeq \operatorname{Int}\operatorname{Aut} G \rtimes \operatorname{Aut} S(G) \end{aligned}$$

Theorem (Field case[1])

Let K be a Hahn filed with the the first lifting property. Then

$$\begin{aligned} v\text{-}\operatorname{Aut} K &= \operatorname{Int}\operatorname{Aut} K \rtimes \operatorname{Ext}\operatorname{Aut} K \\ &\simeq \operatorname{Int}\operatorname{Aut} K \rtimes (\operatorname{Aut} k \times o\text{-}\operatorname{Aut} G) \end{aligned}$$

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Rayner groups

Let $\mathcal{F}$ be a family of subsets of Γ such that

(R1) The members of $\mathcal{F}$ are well ordered subsets of Γ;

(R2) $A, B \in \mathcal{F} \Rightarrow A \cup B \in \mathcal{F}$;

(R3) $A \in \mathcal{F},\ B \subset A \Rightarrow B \in \mathcal{F}$;

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Theorem (Rayner, 1968)

Let $\mathcal{F}$ satisfy (R1)-(R3). Then $\mathbb{G}(\mathcal{F}) := \{a \in \Gamma : \operatorname{supp}(a) \in \mathcal{F}\}$ is a Hahn group.

Rayner groups

Theorem

Let $G = \mathbb{G}(\mathcal{F})$ be a Rayner group. Then G has the lifting property if and only if $\mathcal{F}$ is stable under the action of $\operatorname{Aut} S(G)$.

Examples

- $\mathbf{H}_{\gamma\in\Gamma} A_\gamma$ and $\coprod_{\gamma\in\Gamma} A_\gamma$
- κ infinite cardinal
  $\mathcal{F}_\kappa$ family of all well ordered subsets of Γ of cardinality smaller than κ.
  $\mathbb{G}_\kappa := \mathbb{G}(\mathcal{F}_\kappa)\{a \in \mathbb{G} : |\operatorname{supp}(a)| < \kappa\}$ is the κ-bounded subgroup of $\mathbb{G}$.

Rayner Fields

Let $k((G))$ be a maximal Hahn field. Let $\mathcal{F}$ be a family of subsets of G such that

(R1) The members of $\mathcal{F}$ are well ordered subsets of G;

(R2) $A, B \in \mathcal{F} \Rightarrow A \cup B \in \mathcal{F}$;

(R3) $A \in \mathcal{F},\ B \subset A \Rightarrow B \in \mathcal{F}$;

(R4) The union of the elements of $\mathcal{F}$ generates G as a group;

(R5) $A \in \mathcal{F},\ g \in G \Rightarrow A + g \in \mathcal{F}$;

(R6) if $A \in \mathcal{F}$ and $A \subseteq G^{\geq 0}$ then the set of all finite sums of elements of A belongs to $\mathcal{F}$.

Theorem (Rayner, 1968)

Let $\mathcal{F}$ satisfy (R1)-(R6). Then $k((\mathcal{F})) := \{a \in \mathbb{K} : \mathrm{supp}(a) \in \mathcal{F}\}$ is a Hahn subfield of $\mathbb{K}$.

L. S. Krapp, S. Kuhlmann, and M. Serra. "On Rayner structures". In: *Communications in Algebra* 50.3 (2022), pp. 940–948. DOI: 10.1080/00927872.2021.1976789

Theorem

Let K be a Hahn filed with the the first lifting property. Then

$$
\begin{aligned}
v\text{-}\operatorname{Aut} K &= \operatorname{Int}\operatorname{Aut} K \rtimes \operatorname{Ext}\operatorname{Aut} K \\
&\simeq \operatorname{Int}\operatorname{Aut} K \rtimes (\operatorname{Aut} k \times o\text{-}\operatorname{Aut} G)
\end{aligned}
$$

Theorem

Let $K = k(\mathcal{F})$ be a Rayner field. Then K has the first lifting property if and only if $\mathcal{F}$ is stable under the action of $o\text{-}\operatorname{Aut} G$.

Corollary

The maximal $k((G))$, minimal $k(G)$, the κ-bounded Hahn fields $\mathbb{K}_\kappa$ have the first lifting property.

Order preserving automorphisms

Let G be an ordered Hahn group with skeleton $S(G) = [\Gamma; A_\gamma]$ and let $\sigma \in o\text{-}\operatorname{Aut} G$.
If all the A_γ's are archimedean the natural valuation induced by the ordering coincides with the canonical valuation introduced above.
Thus σ induces an order preserving automorphism $\sigma_\Gamma \in \operatorname{Aut}(\Gamma, <)$
This gives rise to a group homomorphism

$$\Phi_\Gamma : o\text{-}\operatorname{Aut} G \longrightarrow o\text{-}\operatorname{Aut} \Gamma, \ \sigma \longmapsto \sigma_\Gamma.$$

Let $G = \coprod_{\gamma \in \Gamma} A_\gamma$
Let $\operatorname{End} A_\gamma$ be the ring of endomorphisms of A_γ (with pointwise addition and composition).
For all $\alpha, \beta \in \Gamma$ let $H_{\alpha\beta} = \operatorname{Hom}(A_\beta, A_\alpha)$ be the group of homomorphisms from A_β into A_α (with pointwise addition). Let Δ be the set of all $\Gamma \times \Gamma$-matrices $(\sigma_{\alpha\beta})$ where

(i) $\sigma_{\alpha\alpha} \in \operatorname{End} A_\alpha$;
(ii) $\sigma_{\alpha\beta} \in H_{\alpha\beta}$;
(iii) for every β and for all $a \in A_\beta$ we have $\sigma_{\alpha\beta}(a) = 0$ for all but finitely many α.

Then Δ forms a ring with respect to the usual matrix addition and multiplication (condition (iii) ensures that the product be well defined).

Proposition

There is a ring isomorphism $\operatorname{End} G \simeq \Delta$. *Thus, the automorphisms of* G *correspond to the invertible matrices in* Δ.

Correspondence $\Delta \simeq \operatorname{End} G$

for $a = \sum a_\gamma \mathbb{1}_\gamma \in G$ and $(\sigma_{\alpha\beta}) \in \Delta$ we can consider the row vector (a_γ) and multiply it on the left to get

$$(a_\gamma)(\sigma_{\alpha\beta}) = \left(\sum_{\alpha \in \operatorname{supp}(a)} \sigma_{\alpha\beta}(a_\alpha) \right)_{\beta\in\Gamma} =: (b_\beta)_{\beta\in\Gamma}$$

we thus obtain the isomorphism

$$\begin{array}{ccc} \Delta & \longrightarrow & \operatorname{End} G \\ (\sigma_{\alpha\beta}) & \longmapsto & \left(\begin{array}{ccc} G & \longrightarrow & G \\ \sum_{\gamma\in\Gamma} a_\gamma \mathbb{1}_\gamma & \longmapsto & \sum_{\gamma\in\Gamma} b_\gamma \mathbb{1}_\gamma \end{array} \right) \end{array}$$

Characterising order preserving automorphisms

Let T consist of all matrices $(\sigma_{\alpha\beta}) \in \Delta$ such that

(i') $(\sigma_{\alpha\beta})$ is lower triangular;

(ii') for all $\alpha \in \Gamma$, $\sigma_{\alpha\alpha} \in o\text{-}\mathrm{Aut}\, A_\alpha$.

Proposition

A matrix $(\sigma_{\alpha\beta}) \in T$ induces an order preserving endomorphism σ on G. In particular, an invertible matrix in T induces an order preserving automorphism of G.

Characterising order preserving automorphisms

Let U denote the group of units (the invertible matrices) in T.

- U embeds into $o\text{-}\operatorname{Aut} G$.
- All order preserving automorphisms of G induced by the elements $(\sigma_{\alpha\beta}) \in U$ induce the identity on Γ.
- Internal automorphisms corresponds to those matrices such that $\sigma_{\alpha\alpha} = 1$ for all $\alpha \in \Gamma$. If we denote by U^1 the normal subgroup of U consisting of lower uni-triangular matrices, we thus have $\operatorname{Int}\operatorname{Aut} G \simeq U^1$.
- If $U^d < U$ consists of the diagonal matrices in U we then have $U = U^1 \rtimes U^d$.

Characterising order preserving automorphisms

Theorem 2

Let us denote by $o\text{-}\operatorname{Aut}_\Gamma G$ the automorphisms of G that induce the identity on the chain Γ. Similarly, let $\operatorname{Aut}_\Gamma S(G)$ denote the group of automorphisms of the skeleton whose component on Γ is the identity. Then we have

$$o\text{-}\operatorname{Aut}_\Gamma G \simeq \operatorname{Int}\operatorname{Aut} G \rtimes \operatorname{Aut}_\Gamma S(G) \simeq U^1 \rtimes U^d.$$

Next objectives

- Improve the description in Theorem 1 providing a study of $\operatorname{Int}\operatorname{Aut} G$, for a general Hahn group G;
- Improve Theorem 2 by
  - Providing a full study of $o\text{-}\operatorname{Aut} G$, when G is a minimal Hahn group;
  - Extending this to general Hahn groups.

Hints from the Hahn field case – reminders

- Let $(G, +, 0, <)$ be a totally ordered abelian group and k a field.
- Denote by

$$\mathbb{K} := k((G)) := \left\{ a = \sum_{g \in G} a_g t^g : \begin{array}{c} \mathrm{supp}(a) \\ \text{is w.o.} \end{array} \right\}$$

- $k[G] := \{a \in \mathbb{K} : \mathrm{supp}(a) \text{ is finite}\}$;
- $k(G) := \mathrm{Frac}\, k[G]$
- $k(G) \subseteq K \subseteq \mathbb{K}$

- $v \colon K \to G \cup \{\infty\}$

$$v(a) = \begin{cases} \min \mathrm{supp}(a) & a \neq 0 \\ \infty & a = 0 \end{cases}$$

- Valuation ring: $R_K = \{a \in K : v(a) \geq 0\}$
- Valuation ideal: $I_K = \{a \in K : v(a) > 0\}$
- Residue field: $\bar{K} = R_K / I_K \simeq k$

Hints from the Hahn field case – results from [KS22]

First lifting property

$$\begin{array}{ccc} \operatorname{Aut} k \times o\text{-}\operatorname{Aut} G & \longrightarrow & v\text{-}\operatorname{Aut} K \\ (\rho, \tau) & \longmapsto & \left(\sum_{g \in G} a_g t^g \mapsto \sum_{g \in G} \rho(a_g) t^{\tau(g)}\right) \end{array}$$

$$v\text{-}\operatorname{Aut} K \simeq \operatorname{Int}\operatorname{Aut} K \rtimes (\operatorname{Aut} k \times o\text{-}\operatorname{Aut} G)$$

Second lifting property

$$\begin{array}{ccc} \operatorname{Hom}(G, k^\times) & \longrightarrow & \operatorname{Int}\operatorname{Aut} K \\ x & \longmapsto & \left(\sum_{g \in G} a_g t^g \mapsto \sum_{g \in G} a_g x(g) t^g\right) \end{array}$$

$$\operatorname{Int}\operatorname{Aut} K \simeq 1\text{-}\operatorname{Aut} K \rtimes \operatorname{Hom}(G, k^\times)$$

Restricting to strongly linear automorphisms

$$1\text{-}\operatorname{Aut}_k^+ K \simeq (\operatorname{Hom}^+(G, 1 + I_K), \times_S)$$

Further work

- Restrict to strongly additive automorphisms to extend Theorem 2 to general Hahn groups (infinite supports)
- Characterise admissible automorphisms of Γ to transfer the above results to more general valuation preserving automorphisms of Hahn groups.
- ...

Thank you!

Dankeschön!

Grazie!

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