Maxwell equations in models for electromagnetically active solids

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In models for functional materials the equations of thermodynamics are coupled to the Maxwell equations. The coupling appears in the constitutive equations and in the equation expressing the energy balance law. An important step in the model formulation is to derive restrictions for the form of the constitutive equations by inserting the energy balance law into the entropy inequality, hence the energy balance law is also needed at this place.

The energy balance is obtained by adding the thermodynamic energy balance and the electrodynamic energy balance. Yet, it is natural to formulate the thermomechanical equations in material coordinates and the Maxwell equations in Eulerian coordinates, which leads to energy balance equations formulated in material coordinates in the case of thermodynamics and formulated in Eulerian coordinates in the case of electrodynamics. Because of this, the combined energy balance cannot be obtained by simply adding the two separate energy balance equations.

This problem is solved best by transforming the Maxwell equations to material coordinates. We discuss this transformation, which is not a purely mathematical problem, but also employs physical arguments, and we show how the combined energy balance is used in the entropy inequality.

What we present is part of an ongoing larger project, where the formulation of models for functional materials is needed. The presented material is essentially all known, but we hope we could streamline the material and present it in a form prefered by mathematicians.